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SATURATED IMPULSE FOR PULSE-LOADED ELASTIC-PLASTIC SQUARE PLATES

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Abstract—This paper investigates the phenomenon of saturated impulse in elastic-plastic plates whilst a fully clamped square plate is taken as a typical example, and the correlation is made between the saturated impulse obtained from the elastic-plastic analysis and that predicted by the rigid-plastic approximation. The present analysis suggests that two saturated impulses can be defined for an elastic, perfectly plastic structure: one is for the maximum deformation and the other is for the permanent (final) deformation of the structure. In general, the former is smaller than the latter. For the clamped square plates calculated, these two saturated impulses in the high load range lie between the upper and the lower bounds of the saturated impulse obtained from a rigid, perfectly plastic analysis; and finally, a "saturated duration" for loading pulse (critical pulse length) is proposed. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

When a plate is subjected to intense transverse dynamic loading, such as impact of a projectile or a shock wave produced by explosion, it usually undergoes a large elastic-plastic deflection; in some practical problems, the deflection of the plates often exceeds the plate thickness by several times or even more. Consequently, membrane forces are induced by large deflection and greatly enhance the load-carrying capacity of the plates.

Over the past 25 years many efforts have been made to study the effect of membrane forces during the large dynamic deflection of plates. Jones *et al.* (1970, 1972) and Nurick *et al.* (1986) performed experiments on the dynamic plastic behaviour of fully clamped rectangular plates using sheet explosive, for which the ranges of deflection-thickness ratio were 0.1–12.0. Approximate theoretical procedures, which retain the influence of finite deflection and aim to estimate the permanent transverse deflections, were proposed by Jones (1971) and Baker (1975) by using energy balance. In these approaches the material was assumed to be rigid, perfectly plastic and various approximations were made; nevertheless, the method proposed by Jones (1971) gives surprisingly good agreement with the corresponding experimental results.

More recently, Nurick *et al.* (1987) adopted a method in which the mode shape was computed at each time step, allowing the transverse deflection as well as the strain distribution to be predicted. Yu and Chen (1992) then presented a theoretical procedure of tracing the large deflection dynamic response (including the transient plate) of rectangular plates. Their predictions on the final deflection coincide excellently with the experimental data for deflection up to 5–10 times the plate thickness. Zhu (1996a) investigated, both experimentally and numerically, the transient deformation modes of square plates under explosive loading. An optical technique for capturing deformation profiles of square plates

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was demonstrated and evaluated. Elastic-plastic numerical predictions showed good agreement with the experimental results not only of permanent deflection but also of transient deformation profiles.

Because the load-carrying capacity of plates is greatly enhanced by the membrane forces induced by large deflection, if a plate is subjected to a rectangular pressure pulse (or force pulse) with a sufficiently long duration, then only an early part of the pulse contributes to the maximum and permanent deflection of the plate, and the rest of the loading pulse will make no further increase in these deflections. In other words, for a rectangular pulse of fixed loading magnitude, there is a critical pulse length (saturated dimension), beyond which the deformation of plate does not increase. This phenomenon was first explicitly illustrated by Zhao *et al.* (1994a) who analysed the large dynamic plastic deflection of a simply supported beam subjected to a rectangular pressure pulse. These authors (1994b) also developed their idea on the saturated impulse into the cases of circular plates, simply supported and clamped square plates and cylindrical shells.

Noting that the above theoretical investigations on saturated impulse are limited by the rigid, perfectly plastic idealization, the present paper aims first to examine the existence of a saturated impulse for elastic-plastic plates whilst a fully clamped square plate is taken as a typical example, and then to explore the correlation between the saturated impulse obtained from the elastic-plastic analysis and that predicted by the rigid-plastic approximation. For easy use in engineering design, both expressions for saturated impulse and saturated duration (critical pulse length) are provided.

2. RIGID PLASTIC ANALYSIS

Cox and Morland (1959) derived an exact rigid-plastic solution for a special case of a simply-supported square plate subjected to a uniformly distributed dynamic load which produces small deflection. From the rigid perfectly plastic analysis, the loading pressure can be classified as low, medium and high loads according to its amplitude p_0 in comparison with the static plastic collapse load p_c; that is, the load ($p_0 < p_c$) which causes no plastic deformation is termed low load; the load ($p_c < p_0 < 2p_c$ in the case of square plates), which produces a deformation mode identical to the static collapse mechanism is termed the medium load; and the load ($p_0 > 2p_c$) which produces a time-dependent deformation mechanism in the transient phase is termed the high load. It is evident that Cox and Morland's solution can be applied to rectangular plates subjected to a medium load. However, due to the complexity of the generalised stresses at finite deflection, it is difficult to find exact solution for a square or rectangular plate subjected to high load.

Based on the rigid-plastic idealisation, Jones (1971) studied the dynamic plastic response of beams and plates with finite deflections and developed an approximate theoretical procedure to estimate the permanent deflections. Jones adopted a time-independent deformation mode used by Wood (1961) on the ground that the experimental results showed the permanent deformed profiles of rectangular plates loaded dynamically are similar to the shape of the Wood's velocity field. Detailed analysis was given for the particular case of a fully clamped rectangular plate under an uniformly distributed dynamic pressure pulse. The approximate procedure provides a simple but useful tool in predicting the maximum plastic deformation of plate under pulse loading.

For square plate which obeys a circumscribing square yield conditions, $M = M_0$ and $N = N_0$ (see Fig. 1) the maximum deflection at the plate centre can be expressed as

$$\frac{W_m}{H} = \sqrt{1+2\eta\left(1-\cos\left(\sqrt{2\frac{\eta-1}{\eta}}\bar{I}\right)\right)}-1 \quad (1)$$

where $\bar{I} = p_0 t_0/\sqrt{\mu H p_c}$, $\eta = p_0/p_c$ and $p_c = 48M_0/a^2$. t_0 is the duration of the pressure pulse, μ the mass per unit area of plate, H and a are thickness and side length of the plate, respectively, and M_0 the fully plastic bending moment.

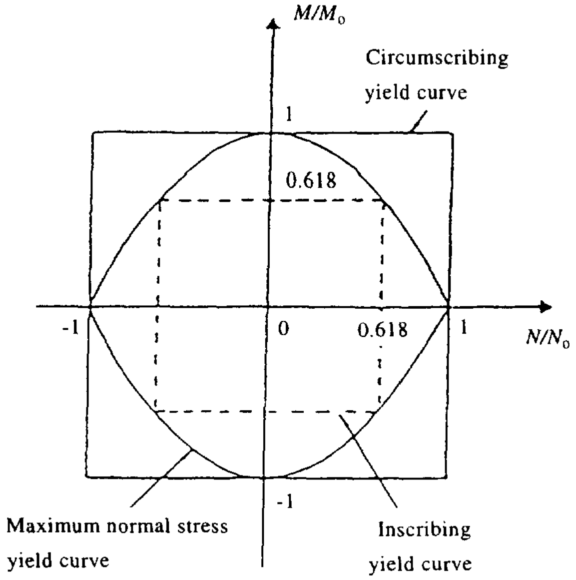

Fig. 1. Yield curves.

In eqn (1), W_m/H is a function of the nondimensional impulse $\bar{I}$ and reaches its maximum when

$$\frac{\sqrt{2}\bar{I}}{\eta} = \pi \tag{2}$$

$$\bar{I}_{sat}^{-} = \frac{\pi}{\sqrt{2}}\eta = 2.22\eta \tag{3}$$

where the superscript $-$ denotes a lower bound which corresponds to the circumscribing square yield condition.

When $\bar{I} < \bar{I}_{sat}^{-}$, W_m/H increases with value $\bar{I}$ and has the maximum value at $\bar{I}_{sat}^{-} = \pi/\sqrt{2}\,\eta$. With $\bar{I} > \bar{I}_{sat}^{-}$ the central deflection in eqn (1) will decrease from the saturated value and fluctuate afterwards. The condition of using eqn (1) is the maximum deflection does not decrease in a deformation process due to deformation irreversibility for a rigid plastic structure. Consequently, the plate has to remain in the saturated state when the applied impulse is larger than the saturated impulse. The additional impulse no longer makes contribution to the maximum deflection of the plate.

Similarly, by adopting an inscribing square yield curve (see Fig. 1), a upper bound value of saturated impulse is obtained as follows

$$\bar{I}_{sat}^{+} = \frac{1}{\sqrt{0.618}}\frac{\pi}{\sqrt{2}}\eta = 2.83\eta. \tag{4}$$

3. ELASTIC-PLASTIC ANALYSIS

The tool employed in this paper to analyze the elastic-plastic response of plate is a numerical program for clamped rectangular plates (Zhu, 1990). It is developed using the Variational Finite Difference Method (VFDM) whilst the effects of elasticity, finite transverse deformation and material strain hardening are included.

This numerical analysis is based on a minimum principle in dynamic plasticity (Lee and Ni, 1973). This assumes that the true acceleration field, $\ddot{U}_k = D^2U_k/DT^2$, of the body which has a known or predetermined displacement and velocity field at time T, is distinguished from all kinematically admissible ones by having the minimum value of the following functional

$$J = \int_{V_0} s^{ij}\ddot{E}_{ij}\,\mathrm{d}\bar{V}_0 + \frac{1}{2}\int_{V_0} \rho_0 \ddot{U}^k \ddot{U}_k\,\mathrm{d}\bar{V}_0 - \int_{S_T} T^k \ddot{U}_k \mathrm{d}S - \int_{V_0} \rho_0 F^k \ddot{U}_k\,\mathrm{d}\bar{V}_0 \tag{5}$$

in which ρ_0 is the initial mass density. In calculation of above functional, the strain energy due to transverse shear is not included, and this is justifiable for thin plates.

The Lagrangian strain may be expressed as the sum of two parts

$$\dot{E}_{ij} = \dot{E}^e_{ij} + \dot{E}^p_{ij}. \tag{6}$$

The elastic stress–strain relationship obeys Hooke's law

$$\dot{E}^e_{ij} = \frac{1}{E}[(1+\nu)\dot{s}_{ij} - \nu\delta_{ij}\dot{s}_{kk}] \tag{7}$$

where ν is Poisson's ratio and δ_{ij} is the Kronecker symbol.

The plastic strain rate tensor can be expressed as follows according to Drucker's postulate,

$$\dot{E}^p_{ij} = G\frac{\partial f}{\partial s_{ij}}\frac{\partial f}{\partial s_{kl}}\dot{s}_{kl}. \tag{8}$$

For an isotropic hardening material, von Mises yield function is adopted and an elastic, linearly strain hardening (bi-linear) stress–strain relation is used in the numerical analysis.

As the central deflection may oscillate about a certain value during the elastic spring-back, it is difficult to determine a permanent deflection of an elastic-plastic plate. Artificial structural damping is introduced after the plate reached its maximum deflection to speed up convergence. In the numerical analysis the damping effects are simulated by adopting resistant pressure over the whole surface of the plate. The magnitude of the resistant pressure is proportional to the damping coefficient and the velocity of the plate.

After discretisation every term in functional J is replaced by discrete values through a central finite difference scheme. Thus, the functional J is replaced by a finite summation by using the 'trapezoidal rule' for integration. With the calculus of variations, the expressions for the accelerations at every nodal point at any time step, $t = q\Delta t$, are obtained by minimising the functional, J, with respect to the discrete accelerations as

$$\frac{\partial J^q}{\partial \ddot{u}^q_{ij}} = 0; \quad \frac{\partial J^q}{\partial \ddot{v}^q_{ij}} = 0; \quad \frac{\partial J^q}{\partial \ddot{w}^q_{ij}} = 0. \tag{9}$$

We then have the expression for $\ddot{u}^q_{ij}$, $\ddot{v}^q_{ij}$ and $\ddot{w}^q_{ij}$ by solving eqn (9). The displacement at time $t+\Delta t$ can be obtained by the central difference approximation

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}^{q+1}_{ij} = \begin{Bmatrix} \ddot{u} \\ \ddot{v} \\ \ddot{w} \end{Bmatrix}^{q}_{ij} (\Delta t)^2 + 2\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}^{q}_{ij} + \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}^{q-1}_{ij}. \tag{10}$$

A time-marching technique is employed to solve the equations governing the dynamic response of the plate. The program has been checked against the experimental results reported by Jones *et al.* (1970) for explosively loaded rectangular plates. For all the 19 aluminium alloy and 22 mild steel plate specimens tested, a surprisingly good agreement has been found for the permanent deflections (Zhu, 1996a). This numerical model was used to analyse the dynamic inelastic response of rectangular plates under rigid wedge impact (see, e.g., Zhu and Faulkner, 1991; Zhu, 1996b) with particular reference to minor ship collision (Zhu and Faulkner, 1994), and to study the transient deformation phenomenon of square plates subjected to explosive loadings (Zhu, 1996a). In both cases, the corresponding experimental results were presented and showed excellent agreement with the numerical predictions.

4. NUMERICAL EXAMPLES

The above numerical program is used to study the saturation behaviour of a square plate subjected to a rectangular pressure pulse (Figs 2 and 3). The geometrical and material parameters in the input data are listed below:

side length of plate	$a = 150$ mm
thickness of plate	$H = 1.5$ mm
density	$\rho = 7800$ kg/m^3
Young's modulus	$E = 207$ GPa
tangent modulus	$E_t = E/1000$
yield stress	$\sigma_y = 210$ MPa.

The following amplitudes of the applied pressure are examined in the analysis

$$\eta = p_0/p_c = 1.33, 2.00, 2.67, 4.00 \text{ and } 6.67.$$

For each pressure amplitude, a series of loading durations (t_0) are selected. The maximum deflection W_m and permanent deflection W_f are calculated by the numerical program for a given pressure pulse (p_0, t_0).

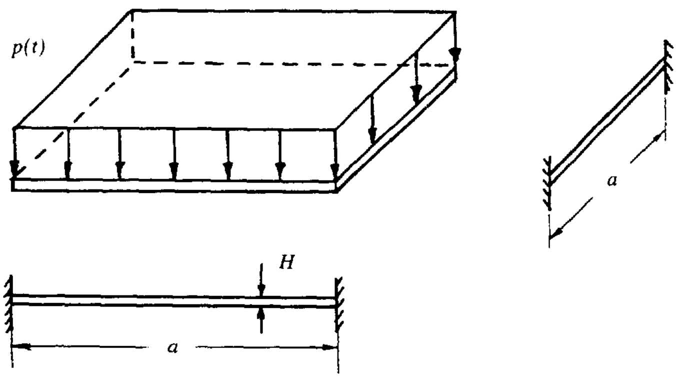

Fig. 2. Fully clamped square plate subjected to uniformly distributed pressure pulse.

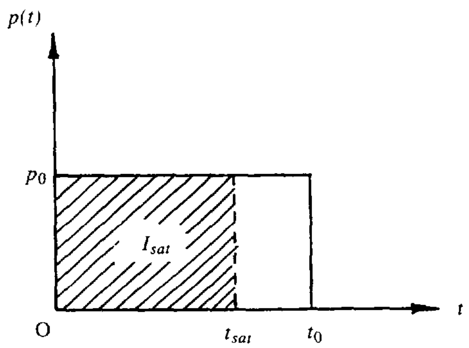

Fig. 3. Rectangular pressure pulse.

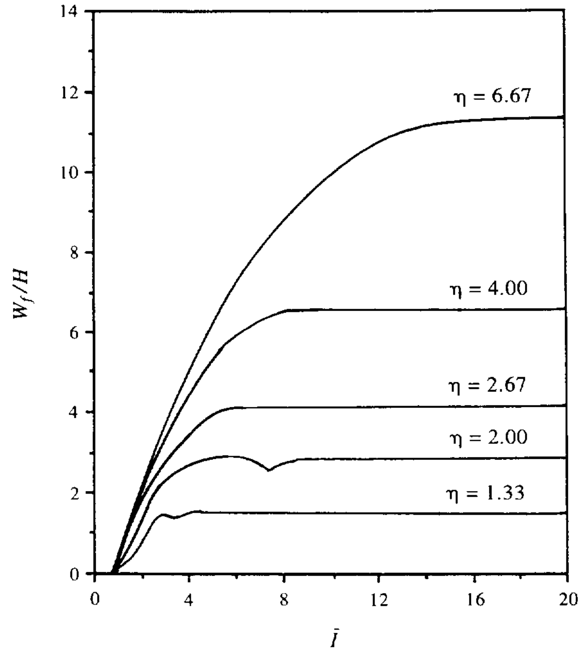

Fig. 4. Dimensionless permanent deflection and impulse relation for various pressure amplitudes.

It is relatively easy to determine the maximum deflection and the associated impulse. The values of W_m and $\bar{I}^m_{sat}$ can be obtained with one run of program if the loading duration t_0 given is sufficiently long relative to the time (t_m) for plate to reach the maximum deflection ($t_0 > t_m$). However, the determination of permanent deflection and its saturated impulse appears more difficult because the elastic recovery and the subsequent loading following the saturation of maximum deflection affect the permanent deflection. Artificial damping is then introduced after the maximum deflection to make the plate approach its permanent state more quickly. In the numerical calculation, the saturation of permeant deflection is determined when the difference between two consecutive runs of program, with close values of t_0 was less than one per cent.

To avoid unexpected interference arising from the introduction of artificial damping, numerical simulation has been performed both with and without damping. In evaluation of the permanent deflection obtained without damping, an average of fluctuated deflections is adopted.

The numerical results are plotted in Fig. 4 for the nondimensional permanent deflection and impulse relation for various pressure amplitudes. In Fig. 4 the curves with $\eta = 2.67$, 4.00 and 6.67 are obtained from the simulation with damping which are almost identical to their no damping counterparts. However, the curves with $\eta = 1.33$ and 2.00 are plotted using the simulation results without artificial damping. It is shown that in the medium load range the permanent deflection may drop at certain pressure impulse after the maximum deflection is saturated, $\bar{I} > \bar{I}^m_{sat}$. This peculiar phenomenon is associated with the elastic recovery of the plate.

It should be mentioned that a unique feature in elastic-plastic analysis is the threshold impulse shown in Fig. 4. Although the rigid-plastic analysis predicts that plastic deformation must occur if $p_0 > p_c$, our elastic-plastic analysis indicates that besides this condition, a certain amount of impulse is required to produce plastic deformation. The threshold impulse is so defined that any impulse below this value would not produce permanent deformation. For the plate examined in this paper, the threshold impulse values for the five given pressure amplitudes are less than $\bar{I} = 1$ (see Fig. 4).

The relationship between the saturated impulse and the pressure amplitude is plotted in Fig. 5, which is obtained from the elastic-plastic analysis and the rigid-plastic method presented in Section 2. The saturated impulses of permanent deflection for $\eta = 1.33$ and 2.00 are not plotted in Fig. 5 due to the peculiar phenomenon mentioned above.

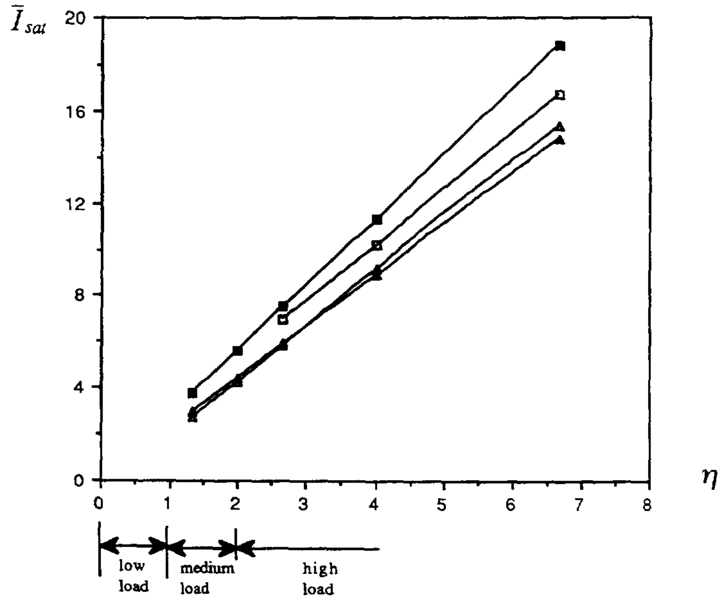

Fig. 5. Variation of nondimensional saturated impulse with nondimensional pressure amplitude predicted by elastic-plastic program and rigid-plastic method: ■, for upper bound maximum deflection (rigid-plastic method); ▲, for lower bound maximum deflection (rigid-plastic method); □, for permanent deflection (elastic-plastic method); △, for maximum deflection (elastic-plastic method).

5. DISCUSSIONS

As demonstrated by the numerical examples in Section 4, the present analysis has confirmed the existence of the saturated impulse for the elastic-plastic response of square plates under rectangular pressure pulse loading regardless of the loading pressure being medium or high. However, in the elastic-plastic analysis, the permanent deflection, W_f of a plate is obviously different from its maximum deflection, W_m. Hence, with regard to the maximum deflection and with regard to the permanent (final) deflection, the saturated impulse has to be defined separately; that is, in general $\bar{I}^m_{sat}$ is different from $\bar{I}^f_{sat}$.

In fact, it has been found from the numerical examples that for the clamped square plates the maximum deflection is saturated first, i.e., $\bar{I}^m_{sat} < \bar{I}^f_{sat}$. This may be explained as follows. Even when the maximum deflection has been saturated, the remaining impulse will continuously disturb the later phase of the response (i.e., after W reaches W_f) and affect the final deflection of the plate. Therefore, the final deflection may only be saturated at a larger impulse.

As observed from Fig. 4, if the loading impulse is between $\bar{I}^m_{sat}$ and $\bar{I}^f_{sat}$ (i.e., the maximum deflection has been saturated but the final deflection has not), the permanent deflection may drop with the increase of impulse within a limited range of impulse. This peculiar phenomenon is associated with the complicated interaction between the remaining impulse and the elastic recovery of the plate. It may be noted that this peculiar phenomenon occurs in the range of energy ratio $R \equiv E_{in}/E_e^{max}$ between 2 and 3, where E_{in} denotes the input energy resulting from the loading pulse and E_e^{max} denotes the maximum elastic deformation energy that can be stored in the structure. A previous study by Yu (1993) has pointed out that the so-called anomalous phenomenon in the dynamic response of a pin-ended elastic-plastic beam (see, e.g., Symonds and Yu, 1985) also occurs in this range of energy ratio. Evidently, the unstable feature of the final deflection in this energy range is somehow similar to the anomalous behaviour studied here.

By employing the elastic-plastic analysis program illustrated in Section 3, the saturated impulses $\bar{I}^m_{sat}$ and $\bar{I}^f_{sat}$ both can be determined, provided the amplitude of loading pulse, p_0, has been specified. Figure 5 shows the relationship of the saturated impulses and the pressure amplitude, and compares the results from the rigid-plastic analysis and the elastic-plastic analysis. It is seen that all of the results exhibit linear relations between the saturated

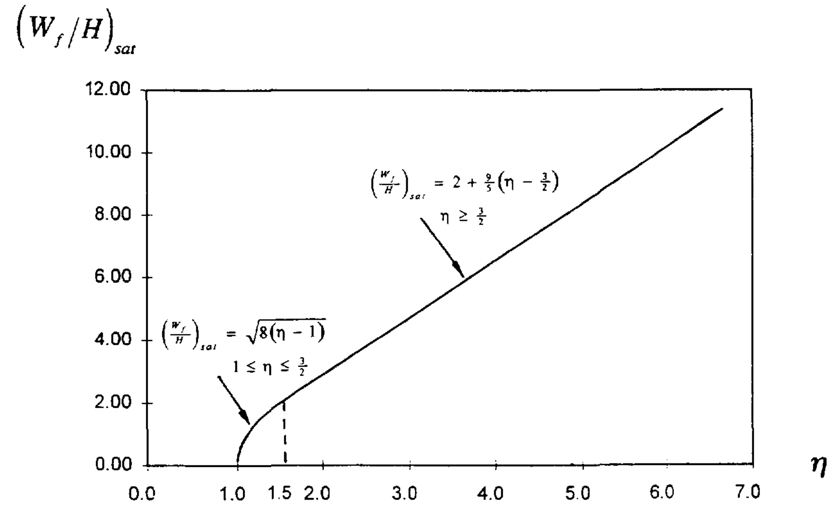

Fig. 6. The variation of nondimensional saturated permanent deflection with nondimensional pressure amplitude.

impulse $\bar{I}_{sat}$ and parameter $\eta = p_0/p_c$; and the lower bound curve based on the rigid-plastic analysis is close to the elastic-plastic prediction of $\bar{I}^m_{sat}$. In the range of high load, say $\eta > 3$, both $\bar{I}^m_{sat}$ and $\bar{I}^f_{sat}$ predicted by the elastic-plastic analysis lie between the upper and lower bound curves obtained from the rigid-plastic analysis.

To provide useful information for use of engineering design, the nondimensional saturated permanent deflection obtained from elastic-plastic analysis is plotted as a function of load amplitude η in Fig. 6. It is obvious that in the medium load range ($p_c < p_0 < 2p_c$) the curve in Fig. 6 exhibits strong non-linearity, but it changes to a straight line in the high load region ($p_0 > 2p_c$). Given an applied pressure amplitude $p_0 = \eta p_c$, the saturated permanent deflection of the plate can be predicted using the curve in Fig. 6. As compared with the upper and lower bounds from the rigid-plastic analysis, Fig. 6 which is based on the elastic-plastic numerical simulation gives an unique and accurate solution.

Although Fig. 6 can be used in the prediction of the saturated deflection, it is not clear under what condition it can be used, in other words, when the plate's deflection would be saturated. It has been noticed in Fig. 5 that the $\bar{I}^f_{sat} - \eta$ relation can be approximated by a straight line passing the origin, as expressed by:

$$\bar{I}^f_{sat} = 2.5\eta. \tag{11}$$

The nondimensional impulse was defined as

$$\bar{I} = \frac{p_0 t_0}{\sqrt{Hp_c}}. \tag{12}$$

We have the following equation from eqns (11) and (12)

$$\frac{p_0 t^f_{sat}}{\sqrt{\mu H p_c}} = 2.5\frac{p_0}{p_c}. \tag{13}$$

Hence

$$t^f_{sat} = 0.72a\sqrt{\frac{\rho}{\sigma_s}}. \tag{14}$$

Here the critical pulse length t^f_{sat} is defined as the saturated duration for the permanent deflection of the square plate.

The saturated duration is the loading time beyond which no further deformation will be produced by the applying pressure pulse. It is important to note from eqn (14) that the saturated duration is a function of geometry and material properties of the plate, and is independent of pressure magnitude. It should be mentioned that for an elastic-plastic structure, the saturated duration is approximate in value as the $\bar{I}^f_{sat}-\eta$ relation from the numerical results is not strictly a straight line passing the origin in Fig. 5. However, the linear relationship between $\bar{I}^f_{sat}$ and η is valid for a rigid-plastic structure, as shown in eqns (3) and (4).

6. CONCLUDING REMARKS

The present analysis suggests that two saturated impulses can be defined for an elastic, perfectly plastic structure: one is for the maximum deformation and the other is for the permanent (final) deformation of the structure. In general, the former is smaller than the latter. For the clamped square plates calculated, these two impulses in the high load range lie between the upper and the lower bounds of the saturated impulse obtained from a rigid, perfectly plastic analysis. For use of engineering design, a $(W_f/H)_{sat}-\eta$ curve is provided based on the elastic-plastic numerical calculations. In exploring the validity of the above design curve, the concept of saturated duration for loading pulse, i.e., critical pulse length, is also proposed which can be used as a thumb of rule in judging the saturation of square plates' deflection.

Some further research topics may be triggered from the present study. For instance, (i) a further study of the unstable feature of the final deflection in a certain range of impulse; (ii) the existence of saturated impulses for beams or plates in case of a non-rectangular pulse loading; (iii) the effect of strain-hardening of materials on the saturated impulses; and (iv) prediction of the threshold impulse of plates.

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